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LIGHT COLOUR-TRIPLET HIGGS IS COMPATIBLE WITH PROTON STABILITY: an alternative approach to the doublet-triplet splitting problem

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Abstract

It is usually assumed that the proton stability requires the coloured triplet partner of the electroweak Higgs doublet to be superheavy (with a mass $\sim M_{GUT}$). We show that this is a very model-dependent statement and the colour triplet can be as light as the weak doublet without leading to the proton decay problem. This implies an alternative approach to the doublet-triplet splitting problem: instead of using the mass difference the splitting can occur between the doublet and triplet Yukawa coupling constants so that the light Higgs triplet can appear decoupled from the quarks and leptons and can not lead to the proton decay. In this scenario the GUT symmetry breaking automatically induces an extremely strong suppression $\sim M_W/M_{GUT}$ of the coloured Higgs effective Yukawa coupling; this happens without any fine-tuning, just because of the Clebsch factors. Conceptual differences of the above picture are: (1) an essentially stable proton: both $d = 5$ and $d = 6$ proton decay mediating operators are suppressed by the same factors $\sim (M_w/M_{GUT})^2$; (2) the possibility of solving the μ problem by the light gauge singlet field (this fact would lead to the destabilization of the hierarchy in the standard case); (3) the existence of the long-lived, light, coloured and charged supermultiplet in the 100 GeV – TeV mass region, which can be the subject of an experimental search. We construct two explicit $SO(10)$ examples with the above properties, with superpotentials most general under the symmetries. In both models, the Higgs sector automatically delivers certain light states which in combination with the coloured triplet form a complete $SU(5)$ multiplet, so that the unification of couplings is unaltered.

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1. Introduction

Perhaps the most difficult problem of the supersymmetric grand unified theories (GUTs) is the ‘doublet–triplet splitting’ problem. The heart of the problem has to do with the fact that in the GUT context the Higgs doublets of the minimal supersymmetric standard model (MSSM) H, \bar{H} inevitably get accompanied by their coloured triplet partners T, \bar{T} . GUT symmetry (G) forces the coloured triplet to be coupled to the quark and lepton superfields $(Q, u_c, d_c, L, e_c, \nu_c)$ by the Yukawa coupling constant $Y_{T(\bar{T})}$, which is equal (in the unbroken G limit) to the one of the doublet $Y_{H(\bar{H})}$. In such a situation the coloured triplet exchange can lead to an unacceptably rapid proton decay unless T, \bar{T} are superheavy. This heaviness is certainly possible since, unless forbidden by some symmetry, the triplet can get large ($\sim M_{GUT}$) mass from the couplings with the vacuum expectation values (VEVs) that break G . What is much, much more difficult, however, is to protect the doublet partners from getting the same order mass. This is the famous doublet–triplet splitting problem. Most of the attempts reported in the literature deal with this difficulty. In certain approaches such as the ‘missing partner’ or the ‘missing VEV’ [1], the doublet appears light for the group theoretical reasons or because of the VEV structure. In the ‘pseudo–Goldstone picture’ [2] its mass is protected by the Goldstone theorem and is fully controlled by the scale of SUSY breaking. The common feature of these approaches is that they try (though in different ways) to make the colour triplet very heavy in order to suppress the proton decay.

Certainly there is a loophole that may avoid such an approach: there is no need for the heavy triplet if its effective Yukawa coupling constant Y_T is suppressed by many orders of magnitude with respect to the one of the doublet Y_H . Say, if $Y_T/Y_H \sim M_W/M_{GUT}$, such a triplet can never lead to an observable proton decay even if its mass is comparable with the weak scale $M_W \sim 100$ GeV–TeV. As suggested in [3], such a situation can occur without any fine–tuning if the matter fermion masses are originated from the effective high–dimensional operators induced by the physics at M_G . To be more explicit consider the following $SO(10)$ invariant operator [3]

$$\frac{Y_{\alpha,\beta}}{M} 10_i 45_{ik} 16^\alpha \gamma_k 16^\beta, \quad (1)$$

where $16^\alpha \alpha = 1, 2, 3$ are three families of the matter fermions, 10_i ($i = 1, \dots, 10$) is the multiplet in which reside the $H, \bar{H} \in 10_i$ ($i = 7, \dots, 10$) and $T, \bar{T} \in 10_i$ ($i = 1, \dots, 6$) states and 45 is the GUT Higgs in the adjoint representation of $SO(10)$. We have written $SO(10)$ tensor indices (i, k) explicitly, since the way of their construction is important for us and γ_i are the matrices of the $SO(10)$ Clifford algebra. M is a certain regulator scale $\sim M_{GUT}$. Coupling (1) has to be understood as an effective operator obtained through the integrating out of some heavy states at M_{GUT} . Below we show explicitly how the above structure can result automatically from the tree–level exchanges [4] of heavy ‘scalar’ or ‘fermionic’ superfields with purely *renormalizable* interactions. Before doing this, let us simply assume for a moment that coupling (1) exists due to whatever reason and see its role in the D–T splitting problem. For this we require the 45-plet Higgs to have the VEV of the form

$$\langle 45_{ik} \rangle = \text{diag}[0, 0, 0, A, A] \otimes \epsilon \quad (2)$$

where $A \sim M_{GUT}$ and each element is assumed to be proportional to the 2×2 antisymmetric matrix ϵ . This VEV breaks the $G_{L,R} = SU(2)_L \otimes SU(2)_R \otimes SU(4)$ subgroup of $SO(10)$ down to $SU(2)_L \otimes U(1)_R \otimes SU(4)$ and is thus oriented along the T_R^3 generator of $SU(2)_R$. In combination with the other VEVs, say the 16-plet with non-zero $SU(5)$ -singlet VEV (ν_c), it leads to the desired breaking $SO(10) \rightarrow G_W = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Now, inserting (2) in (1), we do not need much effort to be convinced that the effective Yukawa coupling constants of the Higgs doublets are $Y_{\alpha,\beta}^H = Y_{\alpha,\beta} \frac{A}{M}$, whereas the triplets have no couplings at all! Such a decoupled triplet cannot lead to the unacceptable proton decay, even if it is as light as its doublet partner. Thus, there is no need to couple the 10-plet to the GUT Higgses and the unnatural fine tuning can be avoided. Of course, in this approach the standard coupling $16^\alpha 16^\beta 10$ must be forbidden by some symmetry. Therefore, we see that the 10-plet must transform under a certain symmetry G_{10} such that: (1) it prevents the 10 from coupling with the GUT Higgses; (2) it allows 10-plet to couple with 16^α *only* in combination with 45-plet. Below we will show that the solution of the μ problem fixes $G_{10} = Z_2 \otimes Z_3$.

What are the motivations for the above approach? First we see at least three model-independent interesting consequences:

(1) It automatically solves the problem of the coloured-Higgsino-mediated proton decay via $d = 5$ operators [5]: In contrast with a standard case the suppression occurs not because of the large mass, but of the small coupling. As a result both $d = 5$ and $d = 6$ operators are suppressed by factors at least $\sim (M_w/M_{GUT})^2$ and the proton is practically stable;

(2) In contrast with the standard GUTs, the μ problem can be solved through the coupling of the Higgs doublet to the light gauge singlet field. In the normal case this would lead to a destabilization of the hierarchy through the well-known ‘tadpole’ diagram [6]. For the obvious reasons, no such diagram exists in the present case.

(3) Prediction of the existence of long-lived T, \bar{T} supermultiplets in the 100 GeV - TeV mass region, which can be the subject of an experimental search.

Last but not least, this approach is an alternative to the existing schemes of the D–T splitting problem and is certainly worth studying, especially since there exists no fully satisfactory solution at present. In this paper we construct the two realistic $SO(10)$ GUTs which explicitly realize the above scenario. Both models have superpotentials most general under symmetries and do not require any fine-tuning or unnaturally small parameters.

2. The mechanism

In this section we study the origin of the operator (1) in somewhat more detail. As was mentioned, it can be generated by the exchange of the intermediate ‘fermionic’ or ‘scalar’ superfields. The reader should not be confused with these names, since they simply refer to the components propagating in the internal line of the tree (super)diagram once we fix the external legs to be fermions for 16^α and scalars for 10 and 45. The ‘fermionic’ exchange was already studied in [3], but we will consider it in detail, since it is an essential ingredient for our models.

2.1 Fermionic exchange

In order to generate (1) through the heavy-fermion exchange we introduce the three pairs of superfields $144_\alpha, \overline{144}_\alpha (\alpha = 1, 2, 3)$, where their representation content under $SO(10)$ is indicated explicitly. The relevant piece of the superpotential has the form

$$W_{Yukawa} = g_\alpha^\beta 16^\alpha \overline{144}_\beta 45 + M^{\alpha\beta} 144_\alpha \overline{144}_\beta + g_\alpha^{'\beta} 16^\alpha 144_\beta 10. \quad (3)$$

This superpotential is invariant under the discrete $Z_2 \otimes Z_3$ symmetry acting in the following way:

under Z_2 : only 10, 45, 144 and $\overline{144}$ change sign;

under Z_3 : $10 \rightarrow e^{i2\theta} 10$; $(16, 144) \rightarrow e^{-i\theta} (16, 144)$, and $\overline{144} \rightarrow e^{i\theta} \overline{144}$.

In fact, any θ leaves W_{Yukawa} invariant, but as shown below only $\theta = 2\pi/3$ is allowed by the solution of the μ problem. This Z_3 symmetry ensures that 10 cannot couple bilinearly with any of the GUT Higgses in the superpotential and both doublet and triplet are light. For energies much below M , this superpotential is effectively equivalent to the operator (1), due to a structure of 144 representation. In order to be sure that the above structure can indeed suppress the proton decay, we have to find the effective couplings of T, \bar{T} with the quarks and leptons. The strategy is straightforward: (1) insert the VEV of 45 in (3); (2) find the light superpositions (express initial states through the final mass eigenstates); (3) insert the answer in the operator $g_\alpha^{'\beta} 16^\alpha 144_\beta 10$ and select only the potentially dangerous couplings $T(\bar{T})$ – light – light. The G_{LR} decompositions of some $SO(10)$ representations are very useful [7]:

$$\begin{aligned} 10 &= (2, 2, 1) + (1, 1, 6) \\ 16 &= (2, 1, 4) + (1, 2, \bar{4}) \\ 144 &= (2, 1, 4) + (1, 2, \bar{4}) + (3, 2, \bar{4}) + (2, 3, 4) + (2, 1, 20) + (1, 2, \overline{20}) \\ 45 &= (1, 3, 1) + \dots \end{aligned} \quad (4)$$

The T, \bar{T} triplets live in $(1, 1, 6)_{10}$. Thus, its G_{LR} -invariant couplings with 16 and 144 are

$$\begin{aligned} g_\alpha^{'\beta} (1, 1, 6)_{10} [& (2, 1, 4)_{16}^\alpha (2, 1, 4)_\beta^{144} + (2, 1, 4)_{16}^\alpha (2, 1, 20)_\beta^{144} \\ & + (1, 2, \bar{4})_{16}^\alpha (1, 2, \overline{20})_\beta^{144} + (1, 2, \bar{4})_{16}^\alpha (1, 2, \bar{4})_\beta^{144}] \end{aligned} \quad (5)$$

We now have to find the light admixture in these fragments. First of all, we immediately notice that there is no state in 16 to which $(2, 1, 4)^{144}$, $(2, 1, 20)^{144}$ and $(1, 2, \overline{20})^{144}$ can mix via $(1, 3, 1)_{45}$ VEV (the would existing states had to transform as $(2, 3, 4)$, $(2, 3, 20)$, $(1, 2, \overline{20})$ and $(1, 4, \overline{20})$, respectively). Thus, $(2, 1, 4)^{144}$, $(2, 1, 20)^{144}$ and $(1, 2, \overline{20})^{144}$ are the purely heavy states and the only potentially dangerous coupling in (4) is the last one. Before discussing its strength, note that this coupling involves only the $SU(2)_R$ -doublet quarks and leptons. Thus, the only possible T -light-light couplings are:

$$\bar{T} u_c d_c + T u_c e_c \quad (6)$$

Even if not suppressed, this couplings can lead to the proton decay only if there is a $T\bar{T}$ mass insertion somewhere. This is not necessary in general, since T, \bar{T} can get masses

from mixing with the other states. The latter can be an interesting possibility *per se*, but it is not necessary in our case, since the above couplings can be naturally absent. To see this, notice that since mixing goes through the VEV $(1, 3, 1)$ the resulting coupling in terms of the light mass eigenstates has the form:

$$(1, 1, 6)_{10} < (1, 3, 1)_{45} > (1, 2, \bar{4})_{light}^\alpha (1, 2, \bar{4})_\beta^{light}. \quad (7)$$

This coupling is antisymmetric in $SU(4)$ indices and symmetric in $SU(2)_R$ ones. So it will automatically vanish if the Yukawa coupling constants are symmetric in α, β . This can be ensured by some flavour symmetry, which sooner or later probably has to be invented anyway in order to solve the fermion-mass problem. As far as we are not going to address this issue here, we will give just one possible example of such a flavour symmetry: $SU(3)_f$, under which 16^α and $144_\alpha, \overline{144}_\alpha$ are antitriplets and triplets, respectively, and which is broken only by the Higgses in the symmetric representation (6-plets). In such a case $g_\alpha^\beta = g\delta_\alpha^\beta$, $g_\alpha^{\prime\beta} = g'\delta_\alpha^\beta$ and $M_{\alpha\beta}$ has to be understood as a VEV of the symmetric representation of $SU(3)_f$.

2.2 Scalar exchange

In order to generate eq(1) via the heavy scalar exchange let us (instead of $144, \overline{144}$) introduce a pair of $10', 10''$ -plets. The superpotential of the Yukawa sector now becomes:

$$W_{Yukawa} = g_{\alpha\beta} 16^\alpha 16^\beta 10'' + g' 45 10' 10 + M 10' 10''. \quad (8)$$

This superpotential is also invariant under a $Z_2 \otimes Z_3$ symmetry such that: under Z_2 , 45 and 10 change sign, and under Z_3 ,

$$16 \rightarrow e^{-i\theta} 16, \quad (10, 10'') \rightarrow e^{i2\theta} (10, 10''), \quad 10' \rightarrow e^{-i2\theta} 10'. \quad (9)$$

After insertion of the 45 VEV the mass matrices of doublets and triplets become

$$(g'AH + MH'')\bar{H}' + (-g'A\bar{H} + M\bar{H}'')H' + M(T'\bar{T}'' + T''\bar{T}'). \quad (10)$$

We see that there is an admixture $\sim g'A/((g'A)^2 + M^2)^{1/2}$ of the light doublet in the H'' state, whereas the light triplet is simply decoupled.

3. Solution of the μ problem

The present approach allows for a simple solution of the μ problem through the introduction of a light gauge singlet superfield N [8]. The VEV of N induced after SUSY breaking plays the role of an effective mass term $\mu H\bar{H}$ in the low-energy theory. The corresponding part of the superpotential has the following form:

$$W_\mu = \lambda N 10^2 + \lambda' N^3/3. \quad (11)$$

This form is the most general under the $Z_2 \otimes Z_3$ symmetry introduced above, provided N is invariant under Z_2 , whereas it transforms in the same way as 10 under Z_3 . In order to

guarantee the decoupling of the N and 10 from the heavy GUT Higgs fields, we require that none of them transform under Z_3 .

Such a simple solution of the μ problem is very difficult to implement in the standard cases with a large D–T mass hierarchy, since the introduction of a light singlet normally leads to a disastrous destabilization of the hierarchy through the well-known one-loop ‘tadpole’ diagram [6]. The same difficulty appears in different versions of the ‘sliding singlet’ scenario [9]. The source of the trouble is that the light singlet couples to both light (H, \bar{H}) and heavy (T, \bar{T}) states; because of this, its exchange immediately induces masses and VEVs of the H, \bar{H} of the order of the geometric mean $\sim (M_{GUT} m_s)^{1/2}$, where m_s is a SUSY-breaking scale in the low-energy sector. This is a serious problem for any scenario (with light singlets) in which the D–T masses are split. In contrast, such a difficulty never occurs in our case, since doublet and triplet are both light ‘by definition’ and N does not couple to the heavy states. So the troublesome ‘tadpole’ is absent, allowing for a simple solution of the μ problem.

It is worth pointing out that even in the standard GUTs with the heavy triplet partner the μ problem may be solved by some other mechanism. For example, when embedded in the minimal supergravity with the hidden sector SUSY breaking [10], the μ problem is automatically solved in the ‘pseudo-Goldstone picture’ [2]; $\mu = m_{3/2}$ is induced by a shift of heavy VEVs triggered by the SUSY breaking. In the other schemes, the solution may be achieved by going beyond the minimal supergravity and introducing the couplings of the 10-plet with the hidden sector fields in the non-minimal Kähler potential [11] (although one may need some effort in order to explain why the similar couplings are absent from the superpotential). However, the solution with light singlet can work equally well even in the schemes with much lower scale of SUSY breaking, where the above supergravity solutions do not work.

4. $SO(10)$ examples

Now, let us turn to the model building and produce two realistic $SO(10)$ examples. The sectors of the theory that are responsible for the proton stability, fermion masses and the μ problem we have discussed above in a more or less model-independent way (apart from the fermion-mass structure, of course, which we believe has to be addressed in the frame of some specific flavour symmetry). In fact what we need now is to take care of the Higgs sector that breaks GUT symmetry and which does not participate in W_{Yukawa} or W_μ due to $Z_2 \times Z_3$ -symmetry. As we know, the only GUT Higgs allowed to speak with W_{Yukawa} is the 45-plet with the VEV (2). Below we will denote it as A . The requirements that the GUT Higgs superpotential (W_{GUT}) has to obey are the following:

- (a) W_{GUT} should be most general under symmetries;
- (b) no ‘fine-tuning’;
- (c) it should allow for the G_w -symmetric SUSY minimum in which A has a VEV (2) and all particles except for one pair of L, \bar{L} -type states + complete $SU(5)$ multiplets + (possibly) some G_W -singlets, have the GUT scale mass in order to keep the successful unification of gauge couplings [12] intact.

We present below two models.

4.1 Model I

W_{GUT} includes the chiral superfields in the following $SO(10)$ representations: $S, X, Y \equiv$ singlets; $\Sigma \equiv 54$ -plet; $A, B, C, \Phi \equiv 45$ -plets; $\chi, \bar{\chi}, \psi, \bar{\psi} \equiv 16, \bar{16}$ -plets and $F \equiv 10$ -plet (not to be confused with the 10-plet in W_{Yukawa}). The superpotential has the form:

$$\begin{aligned}
W_{GUT} = & \frac{\sigma}{4} S Tr \Sigma^2 + \frac{h}{6} Tr \Sigma^3 + \frac{1}{4} Tr (a \Sigma + M_a + a' S) A^2 + \frac{1}{4} Tr (b \Sigma + M_b + b' S) B^2 \\
& + \frac{1}{2} Tr (a'' X A + b'' Y B) C + \frac{g_c}{2} \bar{\chi} C \chi + g_f \chi F \chi + \bar{g}_f \bar{\chi} F \bar{\chi} + g_\Phi \bar{\psi} \Phi \chi \\
& + \bar{g}_\Phi \bar{\chi} \Phi \psi + g_a \bar{\psi} A \psi + \rho X Tr \Phi^2 + M^2 S + \frac{M'}{2} S^2 + \frac{\kappa}{3} S^3
\end{aligned} \tag{12}$$

This form is strictly natural, since it is the most general compatible with the $Z_4^A \otimes Z_2^B \otimes U(1)^C$ global symmetry under which the chiral superfields transform as follows:
under Z_4^A

$$\begin{aligned}
(A, X, 10) & \rightarrow -(A, X, 10) \\
(\psi, \bar{\psi}) & \rightarrow i(\psi, \bar{\psi}) \\
\phi & \rightarrow -i\phi
\end{aligned} \tag{13}$$

under Z_2^B

$$(B, Y) \rightarrow -(B, Y) \tag{14}$$

and under $U(1)^C$

$$\begin{aligned}
(C, F) & \rightarrow e^{i2\alpha}(C, F) \\
(\chi, \bar{\chi}) & \rightarrow e^{-i\alpha}(\chi, \bar{\chi}) \\
(X, Y) & \rightarrow e^{-i2\alpha}(X, Y) \\
\Phi & \rightarrow e^{i\alpha}\Phi
\end{aligned} \tag{15}$$

As the reader can observe, Z_4^A acts as Z_2 on A and 10. This is precisely the same Z_2 symmetry as was introduced in section 2 and which forces *the* 10-plet to be coupled with the matter superfields only in combination with A . We assume that all mass scales in W_{GUT} are $\sim M_{GUT}$ and all coupling constants are of the order of 1.

The standard procedure shows that the above superpotential admits the following supersymmetric (F -flat and D -flat) minimum with an unbroken G_W symmetry:

$$\begin{aligned}
\Sigma &= diag(2, 2, 2, 2, 2, 2, -3, -3, -3, -3) \Sigma \quad \text{where } \Sigma = \frac{b' M_a - a' M_b}{3ab' + 2ba'} \\
A &= diag[0, 0, 0, A, A] \otimes \epsilon \\
B &= diag[B, B, B, 0, 0] \otimes \epsilon \\
\chi &= \bar{\chi} = \chi|+, +, +, +, +\rangle \quad \text{where } \chi^2 = -\frac{a''}{g_c} X A = -\frac{b''}{g_c} Y B \\
S &= -\frac{2bM_a + 3aM_b}{3ab' + 2ba'} \\
\psi &= \bar{\psi} = F = \Phi = C = 0
\end{aligned} \tag{16}$$

According to the standard notations (e.g. see [13]) the $SU(5)$ singlet component of 16 is denoted by $|+, +, +, +, +\rangle$, where each ‘+’ refers to an eigenvalue of the respective Cartan subalgebra generator. The two quantities A and B are determined from the two equations:

$$\begin{aligned} 10(S\sigma\Sigma - h\Sigma^2) - aA^2 + bB^2 &= 0 \\ 15\sigma\Sigma^2 + a'A^2 + \frac{3}{2}b'B^2 + M^2 + M'S + \kappa S^2 &= 0 \end{aligned} \quad (17)$$

Note that the absolute VEVs of the singlets X and Y are undetermined in the SUSY limit, and only their ratio, $\frac{X}{Y} = \frac{b'B}{a''A}$, is. It is not difficult to check that in the given vacuum W_{GUT} delivers a pair of light doublets (with quantum numbers of L, \bar{L}) from the $\psi, \bar{\psi}$ multiplets. This is because $\psi, \bar{\psi}$ states get their masses only from the two sources: through the VEV of A and via mixing with the heavy Φ through $\chi, \bar{\chi}$ VEV. Now, the A VEV leaves all $SU(2)_L$ -doublet states in $\psi, \bar{\psi}$ massless. These are the states with quantum numbers Q, \bar{Q} and L, \bar{L} , which in the $SU(5)$ language belong to $10, \bar{10}$ and $5, \bar{5}$ representations respectively; $10, \bar{10}$ components are mixed with similar fragments of Φ through the $SU(5)$ singlet VEVs of $\chi, \bar{\chi}$ and become heavy. In contrast, $5, \bar{5}$ states cannot do so, since they have no partners in the 45-plet Φ . Thus, L, \bar{L} states are massless. All other G_W non-singlet states from W_{GUT} have a GUT scale mass. If we recall now that in the W_{Yukawa} sector we already had one light triplet pair (T, \bar{T}) on top of the MSSM particle content, it will be clear that new light states form a complete $SU(5)$ -multiplets $(5, \bar{5})$ and the unification of couplings is thus unaltered.

4.2 Model II

In this version the $SO(10)$ content of W_{GUT} is the same as in the Model I except for the fact that we exclude the 10-plet F , the 45-plet Φ , and the two singlets X and Y from the theory. The superpotential becomes:

$$\begin{aligned} W_{GUT} &= \frac{\sigma}{4}STr\Sigma^2 + \frac{h}{6}Tr\Sigma^3 + \frac{1}{4}Tr(a\Sigma + M_a + a'S)A^2 + \frac{1}{4}Tr(b\Sigma + M_b + b'S)B^2 \\ &+ \frac{1}{4}Tr(c\Sigma + M_c + c'S)C^2 + g_a\bar{\psi}A\psi + \frac{1}{2}g_b\bar{\chi}B\chi + \bar{\chi}(M'' + \gamma S)\chi \\ &+ M^2S + \frac{M'}{2}S^2 + \frac{\kappa}{3}S^3 \end{aligned} \quad (18)$$

Again, this form is natural in the strong sense, as it is most general under $Z_4^A \otimes Z_2^C$ symmetry, which acts on the chiral superfields in the following way:

under Z_4^A (as before)

$$(A, 10) \rightarrow -(A, 10), \quad (\psi, \bar{\psi}) \rightarrow i(\psi, \bar{\psi}) \quad (19)$$

under Z_2^C

$$C \rightarrow -C. \quad (20)$$

All other superfields are invariant under the given symmetries. Again, by the straightforward solution of the standard F -flatness and D -flatness conditions we can find the

following G_W -preserving supersymmetric vacuum

$$\begin{aligned}
\Sigma &= \text{diag}(2, 2, 2, 2, 2, 2, -3, -3, -3, -3)\Sigma \quad \text{where } \Sigma = \frac{c'M_a - a'M_c}{3ac' + 2ca'} \\
A &= \text{diag}[0, 0, 0, A, A] \otimes \epsilon \\
B &= \text{diag}[B, B, B, B', B'] \otimes \epsilon \\
C &= \text{diag}[C, C, C, 0, 0] \otimes \epsilon \\
\chi &= \bar{\chi} = \chi|+, +, +, +, + > \\
\psi &= \bar{\psi} = 0 \\
S &= -\frac{2cM_a + 3aM_c}{3ac' + 2ca'} \tag{21}
\end{aligned}$$

The five remaining quantities A, B, B', C , and χ are determined from the five equations:

$$\begin{aligned}
(2b\Sigma + M_b + b'S)B + g_b\chi^2 &= 0 \\
(-3b\Sigma + M_b + b'S)B' + g_b\chi^2 &= 0 \\
\gamma S + M'' + \frac{g_b}{2}(3B + 2B') &= 0 \\
10(\sigma S\Sigma - h\Sigma^2) - aA^2 + b(B^2 - B'^2) + cC^2 &= 0 \\
15\sigma\Sigma^2 + a'A^2 + b(\frac{3}{2}B^2 + B'^2) + \frac{3}{2}c'C^2 + \gamma\chi^2 + M^2 + M'S + \kappa S^2 &= 0 \tag{22}
\end{aligned}$$

Again, in the above vacuum there is a set of G_W non-singlet massless states delivered by W_{GUT} . First of all, there are Q, \bar{Q}, L, \bar{L} states from $\psi, \bar{\psi}$, which are zero eigenstates of T_R^3 generator and cannot get masses from the A VEV. On top of this, there are the pseudo-Goldstone-type massless (in the SUSY limit) states resulting from the continuous degeneracy of the given vacuum. This degeneracy occurs because one can continuously rotate the A and B VEVs by an arbitrary *independent* global $SO(4)$ transformation and/or C and B by an *independent* global $SO(6)$ transformation without violating any of the conditions $F = 0$ or $D = 0$. This happens because A, B , and C do not communicate directly with one another in the superpotential, but only through Σ to whom all 45-plets are coupled bilinearly; since the Σ VEV is invariant under G_{LR} , the vacuum automatically gets a larger degeneracy under $SO(6)_C \otimes SO(6)_B$ and $SO(4)_A \otimes SO(4)_B$ global transformations. Thus, there are pseudo-Goldstone modes with quantum numbers of the $SO(6)/SU(3) \otimes U(1)$ and $SU(2)_R/U(1)_R$ generators ($u_c\bar{u}_c$ and $e_c\bar{e}_c$ states) which are not eaten up by the gauge superfields. Thus again, as in Model I we end up with the complete $SU(5)$ multiplets beyond the MSSM particle content, but now these new light states effectively compose a 4th vector-like family $5 + \bar{5}, 10 + \bar{10}$. This preserves the successful unification of the gauge couplings.

5. conclusions

We have presented an alternative approach to the D–T splitting, problem which in contrast to the standard schemes *does not* require the heavy coloured triplet Higgs. The crucial point is that, independently from the triplet mass, the proton decay can be extremely suppressed for group theoretical reasons if the quark and lepton masses are induced from

the high-dimensional operators of the form (1). In this case, the light coloured triplet *automatically* gets decoupled after the GUT symmetry breaking. We have shown how the desired operators can be naturally (purely due to a symmetry and the field content) induced after integrating out some heavy states at M_{GUT} . Two serious problems of the standard approach: colour-Higgsino-mediated proton decay and the μ problem, can receive a natural solution. The first one is automatic: both Higgsino- and Higgs-mediated proton decays are suppressed by the same rate $(\sim M_W/M_{GUT})^2$ so that the proton is stable (practically). The second problem can be easily solved by introducing a light gauge singlet superfield without causing the standard ‘light singlet’ problem.

Another model-independent consequence is the existence of some decoupled long-lived particles in the low-energy theory. These necessarily include a coloured triplet Higgs pair (and at least one extra doublet pair which automatically preserves the successful unification of couplings). These new particles can be subject of an experimental search. The extremely small Yukawa coupling constant (suppressed at least by a factor $\sim \frac{M_W}{M_{GUT}}$ with respect to the ordinary doublet) makes the lightest member of the supermultiplets T, \bar{T} long-lived enough to appear stable in the detector so that the colour singlet bound states, which they form with ordinary quarks, should behave as heavy (with mass $\sim M_w$) stable hadrons (or mesons). In this respect their phenomenology is very similar to the one of the coloured pseudo-Goldstone states discussed in [14], although their origin is very different.

Finally, we have presented two $SO(10)$ examples which naturally accommodate the above scenario. Both have superpotentials that are most general under symmetries and do not suffer from any fine-tuning problem.

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